PEARSON NEW INTERNATIONAL EDITION

Calculus
Early Transcendentals

C. Henry Edwards David E. Penney

Seventh Edition

ALWAYS LEARNING

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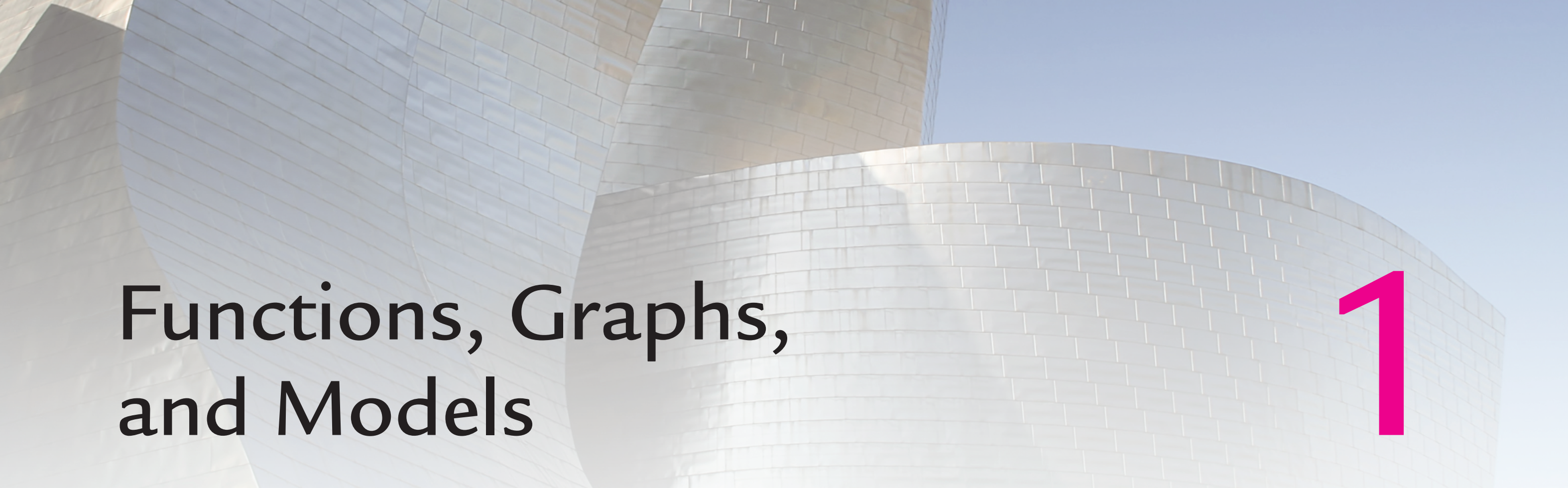


Functions, Graphs, and Models

1

René Descartes (1596–1650)

The seventeenth-century French scholar René Descartes is perhaps better remembered today as a philosopher than as a mathematician. But most of us are familiar with the "Cartesian plane" in which the location of a point P is specified by its coordinates (x, y).

As a schoolboy Descartes was often permitted to sleep late because of allegedly poor health. He claimed that he always thought most clearly about philosophy, science, and mathematics while he was lying comfortably in bed on cold mornings. After graduating from college, where he studied law (apparently with little enthusiasm), Descartes traveled with various armies for a number of years, but more as a gentleman soldier than as a professional military man.

In 1637, after finally settling down (in Holland), Descartes published his famous philosophical treatise *Discourse on the Method* (of Reasoning Well and Seeking Truth in the Sciences). One of three appendices to this work sets forth his new "analytic" approach to geometry. His principal idea (published almost simultaneously by his countryman Pierre de Fermat) was the correspondence between an *equation* and its *graph*, generally a curve in the plane. The equation could be used to study the curve and vice versa.

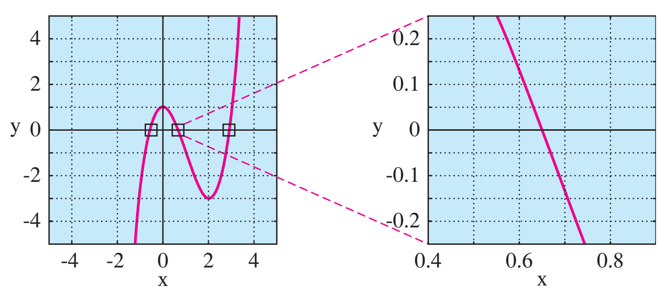
Suppose that we want to solve the equation $f(x) = 0$. Its solutions are the intersection points of the graph of $y = f(x)$ with the x-axis, so an accurate picture of the curve shows the number and approximate locations of the solutions of the equation. For instance, the graph

$$y = x^3 - 3x^2 + 1$$

has three x-intercepts, showing that the equation

$$x^3 - 3x^2 + 1 = 0$$

has three real solutions—one between -1 and 0, one between 0 and 1, and one between 2 and 3. A modern graphing calculator or computer program can approximate these solutions more accurately by magnifying the regions in which they are located. For instance, the magnified center region shows that the corresponding solution is $x \approx 0.65$.



The graph $y = x^3 - 3x^2 + 1$

From Chapter 1 of *Calculus, Early Transcendentals*, Seventh Edition. C. Henry Edwards, David E. Penney.

1.1 FUNCTIONS AND MATHEMATICAL MODELING

Calculus is one of the supreme accomplishments of the human intellect. This mathematical discipline stems largely from the seventeenth-century investigations of Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716). Yet some of its ideas date back to the time of Archimedes (287–212 B.C.) and originated in cultures as diverse as those of Greece, Egypt, Babylonia, India, China, and Japan. Many of the scientific discoveries that have shaped our civilization during the past three centuries would have been impossible without the use of calculus.

The principal objective of calculus is the analysis of problems of change (of motion, for example) and of content (the computation of area and volume, for instance). These problems are fundamental because we live in a world of ceaseless change, filled with bodies in motion and phenomena of ebb and flow. Consequently, calculus remains a vibrant subject, and today this body of conceptual understanding and computational technique continues to serve as the principal quantitative language of science and technology.

Functions

Most applications of calculus involve the use of real numbers or *variables* to describe changing quantities. The key to the mathematical analysis of a geometric or scientific situation is typically the recognition of relationships among the variables that describe the situation. Such a relationship may be a formula that expresses one variable as a *function* of another. For example:

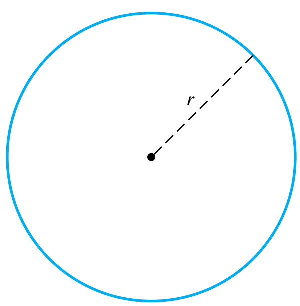


FIGURE 1.1.1 Circle: area $A = \pi r^2$, circumference $C = 2\pi r$.

- The area A of a circle of radius r is given by $A = \pi r^2$ (Fig. 1.1.1). The volume V and surface area S of a sphere of radius r are given by

$$V = \tfrac{4}{3}\pi r^3 \quad \text{and} \quad S = 4\pi r^2,$$

  respectively (Fig. 1.1.2).
- After t seconds (s) a body that has been dropped from rest has fallen a distance

$$s = \tfrac{1}{2}gt^2$$

  feet (ft) and has speed $v = gt$ feet per second (ft/s), where $g \approx 32$ ft/s^2 is gravitational acceleration.
- The volume V (in liters, L) of 3 grams (g) of carbon dioxide at 27°C is given in terms of its pressure p in atmospheres (atm) by $V = 1.68/p$.

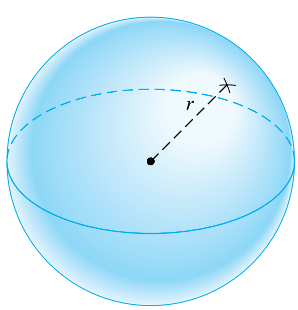


FIGURE 1.1.2 Sphere: volume $V = \frac{4}{3}\pi r^3$, surface area $S = 4\pi r^2$.

DEFINITION Function
A real-valued **function** f defined on a set D of real numbers is a rule that assigns to each number x in D exactly one real number, denoted by $f(x)$.

The set D of all numbers for which $f(x)$ is defined is called the **domain** (or **domain of definition**) of the function f. The number $f(x)$, read "f of x," is called the **value** of the function f at the number (or point) x. The set of all values $y = f(x)$ is called the **range** of f. That is, the range of f is the set

$$\{y : y = f(x) \quad \text{for some } x \text{ in } D\}.$$

In this section we will be concerned more with the domain of a function than with its range.

EXAMPLE 1 The squaring function defined by

$$f(x) = x^2$$

assigns to each real number x its square x^2. Because every real number *can* be squared, the domain of f is the set $\boldsymbol{R}$ of all real numbers. But only nonnegative numbers are squares. Moreover, if $a \geqq 0$, then $a = (\sqrt{a})^2 = f(\sqrt{a})$, so a is a square. Hence the range of the squaring function f is the set $\{y : y \geqq 0\}$ of all nonnegative real numbers.

Functions can be described in various ways. A *symbolic* description of the function f is provided by a formula that specifies how to compute the number $f(x)$ in terms of the number x. Thus the symbol $f(\ )$ may be regarded as an operation that is to be performed whenever a number or expression is inserted between the parentheses.

EXAMPLE 2 The formula

$$f(x) = x^2 + x - 3 \tag{1}$$

defines a function f whose domain is the entire real line $\boldsymbol{R}$. Some typical values of f are $f(-2) = -1$, $f(0) = -3$, and $f(3) = 9$. Some other values of the function f are

$$\begin{aligned} f(4) &= 4^2 + 4 - 3 = 17, \\ f(c) &= c^2 + c - 3, \\ f(2+h) &= (2+h)^2 + (2+h) - 3 \\ &= (4 + 4h + h^2) + (2+h) - 3 = h^2 + 5h + 3, \quad \text{and} \\ f(-t^2) &= (-t^2)^2 + (-t^2) - 3 = t^4 - t^2 - 3. \end{aligned}$$

When we describe the function f by writing a formula $y = f(x)$, we call x the **independent variable** and y the **dependent variable** because the value of y depends—through f—upon the choice of x. As the independent variable x changes, or varies, then so does the dependent variable y. The way that y varies is determined by the rule of the function f. For example, if f is the function of Eq. (1), then $y = -1$ when $x = -2$, $y = -3$ when $x = 0$, and $y = 9$ when $x = 3$.

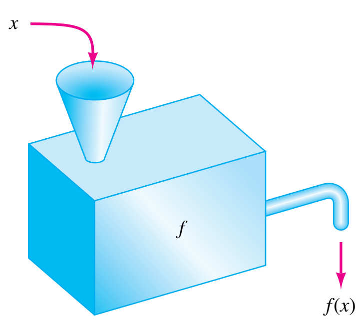


FIGURE 1.1.3 A "function machine."

You may find it useful to visualize the dependence of the value $y = f(x)$ on x by thinking of the function f as a kind of machine that accepts as input a number x and then produces as output the number $f(x)$, perhaps displayed or printed (Fig. 1.1.3).

One such machine is the square root key of a simple pocket calculator. When a nonnegative number x is entered and this key is pressed, the calculator displays (an approximation to) the number $\sqrt{x}$. Note that the domain of this *square root function* $f(x) = \sqrt{x}$ is the set $[0, +\infty)$ of all nonnegative real numbers, because no negative number has a real square root. The range of f is also the set of all nonnegative real numbers, because the symbol $\sqrt{x}$ always denotes the *nonnegative* square root of x. The calculator illustrates its "knowledge" of the domain by displaying an error message if we ask it to calculate the square root of a negative number (or perhaps a complex number, if it's a more sophisticated calculator).

EXAMPLE 3 Not every function has a rule expressible as a simple one-part formula such as $f(x) = \sqrt{x}$. For instance, if we write

$$h(x) = \begin{cases} x^2 & \text{if } x \geqq 0, \\ \sqrt{-x} & \text{if } x < 0, \end{cases}$$

then we have defined a perfectly good function with domain $\boldsymbol{R}$. Some of its values are $h(-4) = 2$, $h(0) = 0$, and $h(2) = 4$. By contrast, the function g in Example 4 is defined initially by means of a verbal description rather than by means of formulas.

EXAMPLE 4 For each real number x, let $g(x)$ denote the greatest integer that is less than or equal to x. For instance, $g(2.5) = 2$, $g(0) = 0$, $g(-3.5) = -4$, and $g(\pi) = 3$. If n is an integer, then $g(x) = n$ for every number x such that $n \leqq x < n + 1$. This function g is called the **greatest integer function** and is often denoted by

$$g(x) = [\![x]\!].$$

Thus $[\![2.5]\!] = 2$, $[\![-3.5]\!] = -4$, and $[\![\pi]\!] = 3$. Note that although $[\![x]\!]$ is defined for all x, the range of the greatest integer function is not all of $\boldsymbol{R}$, but the set $\boldsymbol{Z}$ of all *integers*.

The name of a function need not be a single letter such as f or g. For instance, think of the trigonometric functions $\sin(x)$ and $\cos(x)$ with the names sin and cos.

EXAMPLE 5 Another descriptive name for the greatest integer function of Example 4 is

$$\operatorname{Floor}(x) = [\![x]\!]. \tag{2}$$

(We think of the integer n as the "floor" beneath the real numbers lying between n and $n+1$.) Similarly, we may use $\operatorname{Round}(x)$ to name the familiar function that "rounds off" the real number x to the nearest integer n, except that $\operatorname{Round}(x) = n + 1$ if $x = n + \frac{1}{2}$ (so we "round upward" in case of ambiguity). Round off enough different numbers to convince yourself that

$$\operatorname{Round}(x) = \operatorname{Floor}\left(x + \tfrac{1}{2}\right) \tag{3}$$

for all x.

Closely related to the FLOOR and ROUND functions is the "ceiling function" used by the U.S. Postal Service; $\operatorname{Ceiling}(x)$ denotes the least integer that is not less than the number x. In 2006 the postage rate for a first-class letter was 39¢ for the first ounce and 24¢ for each additional ounce or fraction thereof. For a letter weighing $w > 0$ ounces, the number of "additional ounces" involved is $\operatorname{Ceiling}(w) - 1$. Therefore the postage $s(w)$ due on this letter is given by

$$s(w) = 39 + 24 \cdot [\operatorname{Ceiling}(w) - 1] = 15 + 24 \cdot \operatorname{Ceiling}(w).$$

Domains and Intervals

The function f and the value or expression $f(x)$ are different in the same sense that a machine and its output are different. Nevertheless, it is common to use an expression like "the function $f(x) = x^2$" to define a function merely by writing its formula. In this situation the domain of the function is not specified. Then, by convention, the **domain of the function** f is the set of all real numbers x for which the expression $f(x)$ makes sense and produces a real number y. For instance, the domain of the function $h(x) = 1/x$ is the set of all nonzero real numbers (because $1/x$ is defined precisely when $x \neq 0$).

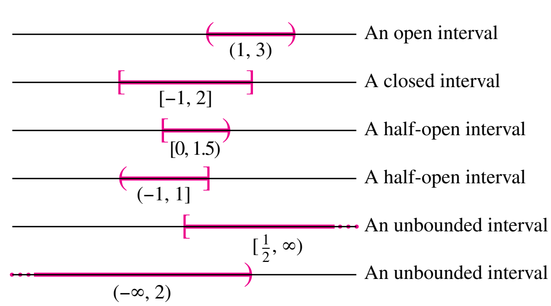


FIGURE 1.1.4 Some examples of intervals of real numbers.

Domains of functions frequently are described in terms of *intervals* of real numbers (Fig. 1.1.4). (Interval notation is reviewed in Appendix A.) Recall that a **closed interval** $[a, b]$ contains both its endpoints $x = a$ and $x = b$, whereas the **open interval** (a, b) contains neither endpoint. Each of the **half-open intervals** $[a, b)$ and $(a, b]$ contains exactly one of its two endpoints. The **unbounded interval** $[a, \infty)$ contains its endpoint $x = a$, whereas $(-\infty, a)$ does not. The previously mentioned domain of $h(x) = 1/x$ is the *union* of the unbounded intervals $(-\infty, 0)$ and $(0, \infty)$.

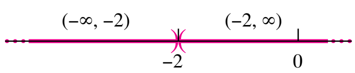


FIGURE 1.1.5 The domain of $g(x) = 1/(2x + 4)$ is the union of two unbounded open intervals.

EXAMPLE 6 Find the domain of the function $g(x) = \dfrac{1}{2x + 4}$.

Solution Division by zero is not allowed, so the value $g(x)$ is defined precisely when $2x + 4 \neq 0$. This is true when $2x \neq -4$, and thus when $x \neq -2$. Hence the domain of g is the set $\{x : x \neq 2\}$, which is the union of the two unbounded open intervals $(-\infty, -2)$ and $(-2, \infty)$, shown in Fig. 1.1.5.

EXAMPLE 7 Find the domain of $h(x) = \dfrac{1}{\sqrt{2x + 4}}$.

Solution Now it is necessary not only that the quantity $2x+4$ be nonzero, but also that it be positive, in order that the square root $\sqrt{2x + 4}$ is defined. But $2x + 4 > 0$ when $2x > -4$, and thus when $x > -2$. Hence the domain of h is the single unbounded open interval $(-2, \infty)$.

Mathematical Modeling

The investigation of an applied problem often hinges on defining a function that captures the essence of a geometrical or physical situation. Examples 8 and 9 illustrate this process.

EXAMPLE 8 A rectangular box with a square base has volume 125. Express its total surface area A as a function of the edge length x of its base.

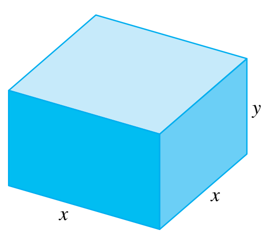


FIGURE 1.1.6 The box of Example 8.

Solution The first step is to draw a sketch and to label the relevant dimensions. Figure 1.1.6 shows a rectangular box with square base of edge length x and with height y. We are given that the volume of the box is

$$V = x^2 y = 125. \tag{4}$$

Both the top and the bottom of the box have area x^2 and each of its four vertical sides has area xy, so its total surface area is

$$A = 2x^2 + 4xy. \tag{5}$$

But this is a formula for A in terms of the *two* variables x and y rather than a function of the *single* variable x. To eliminate y and thereby obtain A in terms of x alone, we solve Eq. (4) for $y = 125/x^2$ and then substitute this result in Eq. (5) to obtain

$$A = 2x^2 + 4x \cdot \frac{125}{x^2} = 2x^2 + \frac{500}{x}.$$

Thus the surface area is given as a function of the edge length x by

$$A(x) = 2x^2 + \frac{500}{x}, \quad 0 < x < +\infty. \tag{6}$$

It is necessary to specify the domain because negative values of x make sense in the *formula* in (5) but do not belong in the domain of the *function* A. Because every $x > 0$ determines such a box, the domain does, in fact, include all positive real numbers.

COMMENT In Example 8 our goal was to express the dependent variable A as a *function* of the independent variable x. Initially, the geometric situation provided us instead with

1. The *formula* in Eq. (5) expressing A in terms of both x and the additional variable y, and
2. The *relation* in Eq. (4) between x and y, which we used to eliminate y and thereby express A as a function of x alone.

We will see that this is a common pattern in many different applied problems, such as the one that follows.

The Animal Pen Problem You must build a rectangular holding pen for animals. To save material, you will use an existing wall as one of its four sides. The fence for the other three sides costs \$5/ft, and you must spend \$1/ft to paint the portion of the wall that forms the fourth side of the pen. If you have a total of \$180 to spend, what dimensions will maximize the area of the pen you can build?

Figure 1.1.7 shows the animal pen and its dimensions x and y, along with the cost per foot of each of its four sides. When we are confronted with a verbally stated applied problem such as this, our first question is, How on earth do we get started on it? The function concept is the key to getting a handle on such a situation. If we can express the quantity to be maximized—the dependent variable—as a function of some independent variable, then we have something tangible to do: Find the maximum value attained by the function. Geometrically, what is the highest point on that function's graph?

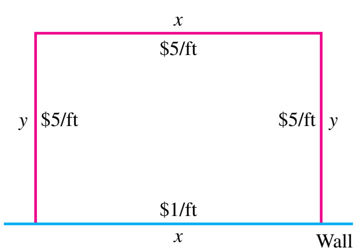


FIGURE 1.1.7 The animal pen.

EXAMPLE 9 In connection with the animal pen problem, express the area A of the pen as a function of the length x of its wall side.

Solution The area A of the rectangular pen of length x and width y is

$$A = xy. \tag{7}$$

When we multiply the length of each side in Fig. 1.1.7 by its cost per foot and then add the results, we find that the total cost C of the pen is

$$C = x + 5y + 5x + 5y = 6x + 10y.$$

So

$$6x + 10y = 180, \tag{8}$$

because we are given $C = 180$. Choosing x to be the independent variable, we use the relation in Eq. (8) to eliminate the additional variable y from the area formula in Eq. (7). We solve Eq. (8) for y and substitute the result

$$y = \tfrac{1}{10}(180 - 6x) = \tfrac{3}{5}(30 - x) \tag{9}$$

in Eq. (7). Thus we obtain the desired function

$$A(x) = \tfrac{3}{5}(30x - x^2)$$

that expresses the area A as a function of the length x.

In addition to this formula for the function A, we must also specify its domain. Only if $x > 0$ will actual rectangles be produced, but we find it convenient to include the value $x = 0$ as well. This value of x corresponds to a "degenerate rectangle" of base length zero and height

$$y = \tfrac{3}{5} \cdot 30 = 18,$$

a consequence of Eq. (9). For similar reasons, we have the restriction $y \geqq 0$. Because

$$y = \tfrac{3}{5}(30 - x),$$

it follows that $x \leqq 30$. Thus the complete definition of the area function is

$$A(x) = \tfrac{3}{5}(30x - x^2), \quad 0 \leqq x \leqq 30. \tag{10}$$

COMMENT The domain of a function is a necessary part of its definition, and for each function we must specify the domain of values of the independent variable. In applications, we use the values of the independent variable that are relevant to the problem at hand.

Example 9 illustrates an important part of the solution of a typical applied problem—the formulation of a **mathematical model** of the physical situation under study. The area function $A(x)$ defined in (10) provides a mathematical model of the animal pen problem. The shape of the optimal animal pen can be determined by finding the maximum value attained by the function A on its domain of definition.

x	$A(x)$
0	0
5	75
10	120
15	135 ←
20	120
25	75
30	0

FIGURE 1.1.8 Area $A(x)$ of a pen with side of length x.

Numerical Investigation

Armed with the result of Example 9, we might attack the animal pen problem by calculating a table of values of the area function $A(x)$ in Eq. (10). Such a table is shown in Fig. 1.1.8. The data in this table suggest strongly that the maximum area is $A = 135$ ft^2, attained with side length $x = 15$ ft, in which case Eq. (9) yields $y = 9$ ft. This conjecture appears to be corroborated by the more refined data shown in Fig. 1.1.9.

Thus it seems that the animal pen with maximal area (costing \$180) is $x = 15$ ft long and $y = 9$ ft wide. The tables in Figs. 1.1.8 and 1.1.9 show only *integral* values of x, however, and it is quite possible that the length x of the pen of maximal area is *not* an integer. Consequently, numerical tables alone do not settle the matter. A new mathematical idea is needed in order to *prove* that $A(15) = 135$ is the maximum value of

$$A(x) = \tfrac{3}{5}(30x - x^2), \quad 0 \leqq x \leqq 30$$

for *all* x in its domain. We attack this problem again in Section 1.2.

x	$A(x)$
10	120
11	125.4
12	129.6
13	132.6
14	134.4
15	135 ←
16	134.4
17	132.6
18	129.6
19	125.4
20	120

FIGURE 1.1.9 Further indication that $x = 15$ yields maximal area $A = 135$.

Tabulation of Functions

Many scientific and graphing calculators allow the user to program a given function for repeated evaluation, and thereby to painlessly compute tables like those in Figs. 1.1.8 and 1.1.9. For instance, Figs. 1.1.10 and 1.1.11 show displays of a calculator prepared to calculate values of the dependent variable

$$y_1 = A(x) = (3/5)(30x - x^2),$$

and Fig. 1.1.12 shows the calculator's resulting version of the table in Fig. 1.1.9.

The use of a calculator or computer to tabulate values of a function is a simple technique with surprisingly many applications. Here we illustrate a method of solving approximately an equation of the form $f(x) = 0$ by *repeated tabulation* of values $f(x)$ of the function f.

As a specific example, suppose that we ask what value of x in Eq. (10) yields an animal pen of area $A = 100$. Then we need to solve the equation

$$A(x) = \tfrac{3}{5}(30x - x^2) = 100,$$

which is equivalent to the equation

$$f(x) = \tfrac{3}{5}(30x - x^2) - 100 = 0. \qquad \textbf{(11)}$$

This is a quadratic equation that could be solved using the quadratic formula of basic algebra, but we want to take a more direct, numerical approach. The reason is that the

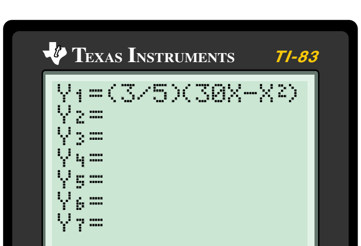


FIGURE 1.1.10 A calculator programmed to evaluate $A(x) = (3/5)(30x - x^2)$.

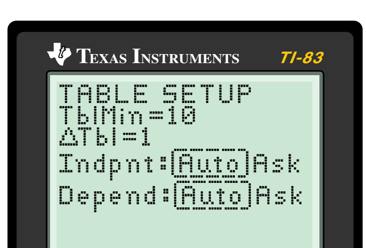


FIGURE 1.1.11 The table setup.

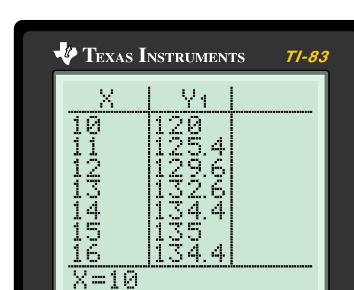


FIGURE 1.1.12 The resulting table.

numerical approach is applicable even when no simple formula (such as the quadratic formula) is available.

The data in Fig. 1.1.8 suggest that one value of x for which $A(x) = 100$ lies somewhere between $x = 5$ and $x = 10$ and that a second such value lies between $x = 20$ and $x = 25$. Indeed, substitution in Eq. (11) yields

$$f(5) = -25 < 0 \quad \text{and} \quad f(10) = 20 > 0.$$

The fact that $f(x)$ is *negative* at one endpoint of the interval [5, 10] but *positive* at the other endpoint suggests that $f(x)$ is *zero* somewhere between $x = 5$ and $x = 10$.

To see *where*, we tabulate values of $f(x)$ on [5, 10]. In the table of Fig. 1.1.13 we see that $f(7) < 0$ and $f(8) > 0$, so we focus next on the interval [7, 8]. Tabulating $f(x)$ on [7, 8] gives the table of Fig. 1.1.14, where we see that $f(7.3) < 0$ and $f(7.4) > 0$.

We therefore tabulate $f(x)$ once more, this time on the interval [7.3, 7.4]. In Fig. 1.1.15 we see that

$$f(7.36) \approx -0.02 \quad \text{and} \quad f(7.37) \approx 0.07.$$

Because $f(7.36)$ is considerably closer to zero than is $f(7.37)$, we conclude that the desired solution of Eq. (11) is given approximately by $x \approx 7.36$, accurate to two decimal places. If greater accuracy were needed, we could continue to tabulate $f(x)$ on smaller and smaller intervals.

If we were to begin with the interval [20, 25] and proceed similarly, we would find the second value $x \approx 22.64$ such that $f(x) = 0$. (You should do this for practice.)

Finally, let's calculate the corresponding values of the width y of the animal pen such that $A = xy = 100$:

- If $x \approx 7.36$, then $y \approx 13.59$.
- If $x \approx 22.64$, then $y \approx 4.42$.

Thus, under the cost constraint of the animal pen problem, we can construct either a 7.36-ft by 13.59-ft or a 22.64-ft by 4.42-ft rectangle, both of area 100 ft^2.

The layout of Figs. 1.1.13 through 1.1.15 suggests the idea of repeated tabulation as a process of successive numerical magnification. This method of repeated tabulation can be applied to a wide range of equations of the form $f(x) = 0$. If the interval $[a, b]$ contains a solution and the endpoint values $f(a)$ and $f(b)$ differ in sign, then we can approximate this solution by tabulating values on successively smaller subintervals. Problems 57 through 66 and the project at the end of this section are applications of this concrete numerical method for the approximate solution of equations.

x	$f(x)$
5	−25.0
6	−13.6
7	−3.4
8	5.6
9	13.4
10	20.0

FIGURE 1.1.13 Values of $f(x)$ on [5, 10].

x	$f(x)$
7.0	−3.400
7.1	−2.446
7.2	−1.504
7.3	−0.574
7.4	0.344
7.5	1.250
7.6	2.144
7.7	3.026
7.8	3.896
7.9	4.754
8.0	5.600

FIGURE 1.1.14 Values of $f(x)$ on [7, 8].

x	$f(x)$
7.30	−0.5740
7.31	−0.4817
7.32	−0.3894
7.33	−0.2973
7.34	−0.2054
7.35	−0.1135
7.36	−0.0218
7.37	0.0699
7.38	0.1614
7.39	0.2527
7.40	0.3440

FIGURE 1.1.15 Values of $f(x)$ on [7.3, 7.4].

1.1 TRUE/FALSE STUDY GUIDE

Use the following true/false items to check your reading and review of this section. You may consult the hints provided in the answer section.

1. Isaac Newton was born in the 18th century.
2. A function is a rule that assigns to each real number in its domain one and only one real number.
3. The value of the function f at the number x in its domain is commonly denoted by $f(x)$.
4. If the domain of the function f is not specified, then it is the set of all real numbers.
5. The function giving the surface area A as a function of the edge length x of the box of Example 8 is given by

$$A(x) = 2x^2 + \frac{600}{x}, \quad 0 \leqq x < +\infty.$$

6. In the animal pen problem (Example 9), the maximum area is attained when the length x of the wall side is 18 ft.
7. The interval (a, b) is said to be open because it contains neither of its endpoints a and b.
8. The domain of $f(x) = \sqrt{x}$ does not include the number $x = -4$.
9. The domain of the function $A(x) = \frac{3}{5}(30x - x^2)$ is the set of all real numbers.
10. There is no good reason why the domain of the animal pen function in Eq. (10) is restricted to the interval $0 \leqq x \leqq 30$.

1.1 CONCEPTS: QUESTIONS AND DISCUSSION

1. Can a function have the same value at two different points? Can it have two different values at the same point x?
2. Explain the difference between a dependent variable and an independent variable. A change in one both causes and determines a change in the other. Which one is the "controlling variable"?
3. What is the difference between an open interval and a closed interval? Is every interval on the real line either open or closed? Justify your answer.
4. Suppose that S is a set of real numbers. Is there a function whose domain of definition is precisely the set S? Is there a function defined on the whole real line whose range is precisely the set S? Is there a function that has the value 1 at each point of S and the value 0 at each point of the real line $\boldsymbol{R}$ not in S?
5. Figure 1.1.6 shows a box with square base and height y. Which of the following two formulas would suffice to define the volume V of this box as a function of y?

   (a) $V = x^2 y$; (b) $V = y(10 - 2y)^2$.

   Discuss the difference between a formula and a function.
6. In the following table, y is a function of x. Determine whether or not x is a function of y.

x	0	2	4	6	8	10
y	−1	3	8	7	3	−2